Neutrino Oscillation and CP Violation

Joe Sato
Research Center for Higher Education, Kyushu University,
Ropponmatsu, Chuo-ku, Fukuoka, 810-8560, Japan.

Abstract

We reconsider the meaning of observing CP violation in neutrino oscillation.

1 Introduction

Many experiments and observations have shown evidences for neutrino oscillation one after another. The solar neutrino deficit has long been observed[1, 2, 3, 4, 5]. The atmospheric neutrino anomaly has been found[6, 7, 8, 9] and recently almost confirmed by SuperKamiokande[10]. All of them can be understood by neutrino oscillation and hence indicates that neutrinos are massive and there is a mixing in lepton sector[11]. Relevant parameters will be determined more precisely in near future[12].

Thus completely unknown parameters for the lepton sector will be

$$\begin{array}{ll} U_{e3} & : \text{Last Mixing} \\ \sin\delta & : \text{CP Violation} \\ \text{sign of } \delta m^2_{\rm atm} & \end{array}$$

in near future.

Then how we determine them is a big problem and it is the main topic of the conference. In this article we will pay attention to CP violating phase.

What energy range is suitable for observing CP violation? Since CP-violation effect arise as three(or more)-generation phenomena[13], we should make an experiment with "not too high" and "not too low" energy to see "3-generation". In an oscillation experiment, there are two energy scales,

$$E \sim \left\{ \begin{array}{l} \delta m^2_{31} L \\ \delta m^2_{21} L \end{array} \right. . \tag{1}$$

Then the above energy range is expected to be suitable for a neutrino oscillation experiment to see CP violation in lepton sector[14].

Indeed in high energy the first two lightest states seem "degenerate".

$$\Longleftrightarrow \delta m^2_{21} \sim 0$$

and hence the oscillation term for CP violation becomes 0:

$$\begin{aligned} & \sin\Delta_{21} + \sin\Delta_{32} + \sin\Delta_{13} \\ \sim \quad & \sin\Delta_{31} + \sin\Delta_{13} \Longrightarrow 0. \end{aligned}$$

In low energy the heaviest (two) state(s) "decouple(s)".

$$\Longleftrightarrow \Delta m^{2'} s \sim \infty$$

and therefore the oscillation term is averaged away within finite resolution for neutrino energy:

$$\begin{array}{lccccc} & \sin\Delta_{21} & + & \sin\Delta_{32} & + & \sin\Delta_{13}. \\ \text{oscillating out} & \downarrow & & \downarrow & & \downarrow \\ & 0 & & 0 & & 0 \end{array}$$

From this consideration,

$$\delta m^2_{21} L \le E \le \delta m^2_{31} L \tag{2}$$

will be the best energy range.

Moreover, to avoid the uncertainty due to matter effect[15], lower energy and shorter baseline length are better.

Experimentally there are two energy region for neutrino experiment[16]. One is $E_\nu \sim$ 0.1-1 GeV and the other $E_\nu > 5$ GeV which is considered extensively in the context of neutrino factory.

2 Oscillation probability $P(\nu_\alpha \to \nu_\beta)$ for $E \sim$ 0.1-1 GeV and $L \sim$ O(100) Km

In this subsection we will consider the neutrino oscillation experiment with $E_\nu \sim$ 0.1-1 GeV. For this energy region the suitable baseline length L to see CP violation is determined to be on the order of 100 km by the "3 generation condition" (1).

For this setting, the transition probability is calculated to be

$$\begin{array}{lcl} & & P(\nu_\mu \to \nu_e) \\ & = & 4|U_{e3}U_{\mu 3}|^2 \sin^2 \dfrac{\Delta_{31}}{2} \\ & + & 4\mathrm{Re}(U^*_{e3}U_{\mu 3}U_{e2}U^*_{\mu 2}) \left(\dfrac{\delta m^2_{21}}{\delta m^2_{31}}\right) \Delta_{31} \sin\Delta_{31} \\ \text{CPV!!} & - & 4\mathrm{Im}(U^*_{e3}U_{\mu 3}U_{e2}U^*_{\mu 2}) \left(\dfrac{\delta m^2_{21}}{\delta m^2_{31}}\right) \Delta_{31} \sin^2 \dfrac{\Delta_{31}}{2} \\ & - & 4\mathrm{Re}(U^*_{e2}U_{\mu 2}U_{e1}U^*_{\mu 1}) \left(\dfrac{\delta m^2_{21}}{\delta m^2_{31}}\right)^2 \left(\dfrac{\Delta^2_{31}}{2}\right)^2 \end{array}$$

$$
\begin{aligned}
&\equiv \quad A \sin^2 \frac{\Delta_{31}}{2} \\
&+ \quad \frac{B}{2} \Delta_{31} \sin \Delta_{31} \\
&+ \quad C \Delta_{31} \sin^2 \frac{\Delta_{31}}{2} \\
&+ \quad D \left(\frac{\Delta_{31}^2}{2} \right)^2
\end{aligned} \tag{3}
$$

up to the leading(second) order of small values,

$$U_{e3},\ \frac{\delta m_{21}^2}{\delta m_{31}^2} \text{ and } \frac{a}{\delta m_{31}^2},$$

here a denotes the matter effect.

There are two comments here: 1) What we can observe are not mixing angles of a certain parameterization but values of certain combination of couplings which are $A - D$ in this case. Without paying attention to this fact, we will not understand correctly the uncertainties on the mixing parameters due to uncertainties of an experiment. 2) For this setting the matter effect, which is the serious obstacle for detecting CP violation, gives only a subleading effect, and in this sense the neutrino energy and the baseline length assumed here seems very preferable. To observe asymmetry in the transition probability means directly the fact that there is a CP violation in lepton sector.

Current bounds on coefficients[17]:

$$
\begin{aligned}
A &\leq 0.05 \\
B &\leq 0.006 \\
C &\leq 0.006 \\
D &\leq 0.001
\end{aligned}
$$

with $U_{e3} \leq 0.15$ and $\frac{\delta m_{21}^2}{\delta m_{31}^2} < 3 \times 10^{-2}$. Due to the different energy dependence, these four terms can contribute the oscillation probability equivalently!!

In eq.(3), base functions,

$$\sin^2 \frac{\Delta_{31}}{2},\ \Delta_{31} \sin \Delta_{31},\ \Delta_{31} \sin^2 \frac{\Delta_{31}}{2},\ \Delta_{31}^2$$

are independent! Since

$$\left\{ \begin{array}{ccc} L & = & 300\text{km} \\ \delta m_{31}^2 & = & 3 \times 10^{-3} \end{array} \right. \Longleftrightarrow \frac{\Delta_{31}}{2} = \left\{ \begin{array}{ccc} \frac{1}{2}\pi & \text{at} & E \sim 700\text{MeV} \\ \frac{3}{2}\pi & \text{at} & E \sim 250\text{MeV} \end{array} \right.$$

in the energy region considered here their behaviors are completely different from each other and hence it is expected that the coefficients $A - D$ are determined rather well.

In fig.2, it is shown how many neutrinos and antineutrinos in detection are necessary to see CP violation when all the mixing parameters except CP

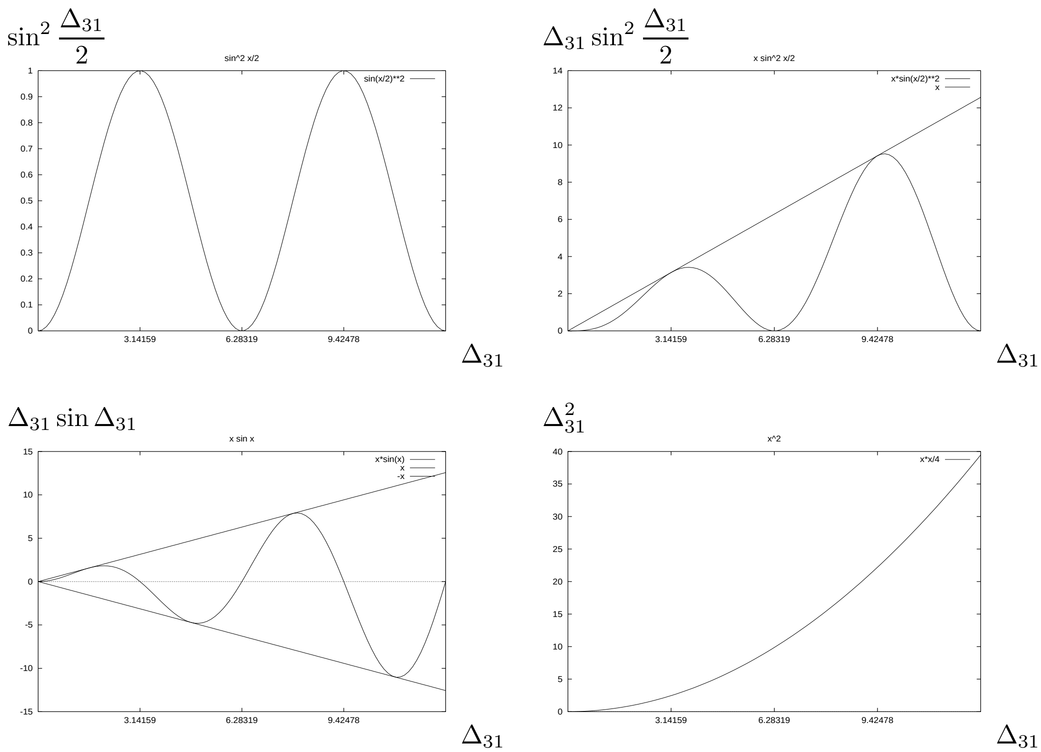

Figure 1: Base functions for transition probability eq.(3).

violating phase are determined precisely as indicated in the caption. According to Konaka[16], for SK size detector (20kt detector) we will have 1000 μ neutrinos in detection per year with the first stage of Japan Hadron Facility(JHF). Then if $\sin\delta$ is large, we can detect CP violation with several years run without before neutrino factory runs.

There is another fruit using the current setting. Though these 4 coefficients seem to be independent, The following relation between coefficients

$$4AD = B^2 + C^2 \tag{4}$$

must be satisfied if there are only 3-generation neutrinos. [1] In other words we may check the unitarity of lepton sector.

[1] Exactly speaking, this relation holds up to $\frac{\delta m_{21}^2}{\delta m_{31}^2}$. If we know the value of $\frac{\delta m_{21}^2}{\delta m_{31}^2}$, then

$$4A\ (\tilde{D} - \frac{\tilde{B}}{2}) = \tilde{B}^2 + \tilde{C}^2,$$

where

$$B = \tilde{B} \times \frac{\delta m_{21}^2}{\delta m_{31}^2}$$

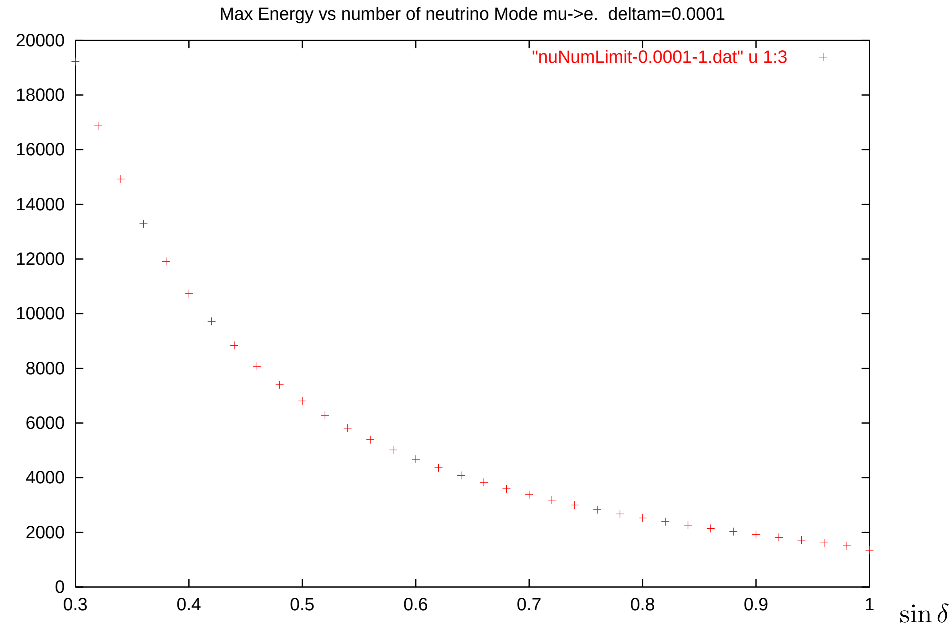

Figure 2: The necessary numbers of neutrinos for observing CP violation as a function of $\sin\delta$ at 99% level. The parameters here are $\delta m^2_{21} = 1\times 10^{-4}$, $\delta m^2_{31} = 3\times 10^{-3}$, , $\sin 2\theta_{12} = 0.8, \sin 2\theta_{23} = 1$ and $\sin 2\theta_{13} = 0.2$.

3 High energy? Low energy?

As noted in the previous section, what we observe are not angles of a certain parameterization but values of certain combination of couplings. We should verify which combination of the couplings are determined well in an oscillation experiment and how we extract CP phase essentially. By this consideration we can understand what kind of uncertainty in experiment affects the uncertainty of determining angles. Here we will consider how CP violation is observed in an oscillation experiment as an example of this idea.

To observe CP violation means to measure the area of the unitarity triangle of the lepton sector. To measure the area there are two ways: 1) Direct measurement. 2) First determining the triangle then calculating it. The first way is strong against other uncertainties, those of other parameters, experiments and so on, since whether there is CP violation is determined by the fact that the area is not 0.

Indeed we have two ways of the determination. The observables $A - D$

$$\begin{aligned} C &= \tilde{C} \times \frac{\delta m^2_{21}}{\delta m^2_{31}} \\ D &= \tilde{D} \times (\frac{\delta m^2_{21}}{\delta m^2_{31}})^2 \end{aligned}$$

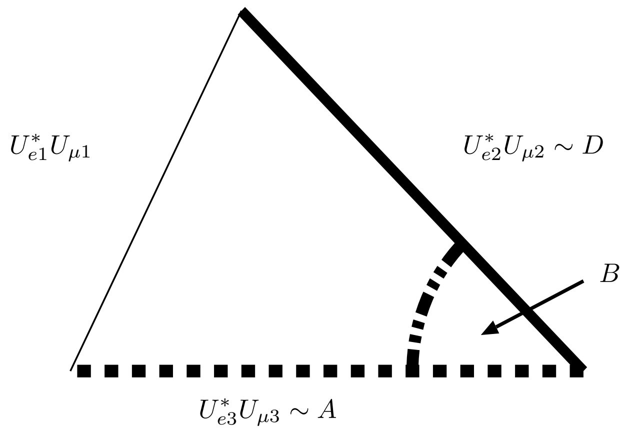

Figure 3: Unitarity triangle for lepton sector. This area shows the strength of CP violation C. The length of the bottom line is determined by A, that of right line corresponds to D and the angle between them is obtained by B.

corresponds the elements of the unitarity triangle indicated in fig.3. C is the area itself. Thus to determine C corresponds to direct measurement. On the other hand, the length of the bottom line is determined by A, that of right line corresponds to D and the angle between them is obtained by B. With these three parameters the triangle are fixed completely and the area is calculated according to the unitarity relation, eq.(4).

Which of "direct measurement" or "unitarity" determines the CP violation essentially?

In higher energy region, the transition probability takes the form,

$$P(\nu_\mu \to \nu_e) = (A + B + D)\Delta_{31}^2 + C\Delta_{31}^3 + \cdots.$$

Thus an experiment in higher energy becomes sensitive to only the combination[2] of $A + B + D$ and the direct CP measure, C, becomes less determined. It means CP violation is measured by the unitarity relation, eq.(4) and hence the determination of CP violation is easily influenced by uncertainty in the experiment.

On the contrary, in lower energy region C is a good observable and hence we can tell whether CP violation is there rather strictly.

With this consideration how important the 3-generation property is. The best energy range for CP violation is in the range given in eq.(2).

[2]Of course, the matter effect distinguishes these observables, though the separation is weaker than the case considered in section 2.

4 Discussion

To see CP violation, we have to see 3-generation of neutrinos simultaneously and hence the energy range, $\delta m_{21}^2 L \leq E \leq \delta m_{31}^2 L.$, is found to be preferable for it. More to say to avoid the matter effect the shorter baseline length is better. Indeed an experimental setup,

$$E \sim \mathrm{O}(100)\mathrm{MeV} \text{ and } L \sim \mathrm{O}(100)\mathrm{km}, \tag{5}$$

is very feasible and much richer information on ν's can be obtained. In this region we can see not only transition but also full oscillation and the observables are the combination of couplings $A - D$ in eq.(3).

Large part of parameter space which will be probed by neutrino factory can be surveyed by the conventional beam with the setup (5). Why don't you consider such a possibility seriously?

Acknowledgments

The author thanks a. Konaka and M. Koike for useful discussions. This research was supported in part by a Grant-in-Aid for Scientific Research of the Ministry of Education, Science and Culture, #12047221, #12740157.

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